Study of a discrete grey forecasting model based on the quality cost characteristic curve

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Abstract

Purpose – The purpose of this paper is to clarify several commonly used quality cost models based on Juran's characteristic curve. Through mathematical deduction, the lowest point of quality cost and the lowest level of quality level (often depicted by qualification rate) can be obtained. This paper also aims to introduce a new prediction model, namely discrete grey model (DGM), to forecast the changing trend of quality cost.

Design/methodology/approach – This paper comes to the conclusion by means of mathematical deduction. To make it more clear, the authors get the lowest quality level and the lowest quality cost by taking the derivative of the equation of quality cost and quality level. By introducing the weakening buffer operator, the authors can significantly improve the prediction accuracy of DGM.

Findings – This paper demonstrates that DGM can be used to forecast quality cost based on Juran's cost characteristic curve, especially when the authors do not have much information or the sample capacity is rather small. When operated by practical weakening buffer operator, the randomness of time series can be obviously weakened and the prediction accuracy can be significantly improved.

Practical implications – This paper uses a real case from a literature to verify the validity of discrete grey forecasting model, getting the conclusion that there is a certain degree of feasibility and rationality of DGM to forecast the variation tendency of quality cost.

Originality/value – This paper perfects the theory of quality cost based on Juran's characteristic curve and expands the scope of application of grey system theory.

Keywords Cost prediction, Quality cost, Discrete grey model, Modelling mechanism

Paper type Research paper

1. Introduction

In the 1950s, J.M. Juran presented a general discussion of quality cost in his *Quality Control Handbook*, in which the cost of poor quality was compared to "the gold in the mine" (Juran, 1951). Over the next several years, Armand V. Feigenbaum first clarified the concept of quality cost and defined it as, "the costs incurred to ensure and guarantee the satisfaction of quality and the loss that did not achieve satisfactory quality". Over the next few decades, there was a growing discussion of quality cost. The models of cost of quality (CoQ) for domestic and foreign scholars mainly included: the prevention-appraisal-failure (PAF) model (Baatz, 1992), the Crosby model (Bemowski, 1991), the opportunity cost model (Bohan and Horney, 1991), the process cost model (Burgess, 1997), the ABC model (Bottorff, 1997) and so on. These models of quality cost lay the foundations for further research on the subject.

For the PAF model, E.B. Baatz divided the total quality cost into prevention cost (P), appraisal cost (A), internal failure cost and external failure cost. Then Juran presented a quality characteristic curve based on this model and he believed that the total mass cost curve was synthesised by the costs of conformance and the costs of non-conformance. The curve of costs of conformance and the curve of costs of non-conformance guaranteed the minimum position of total quality costs: it corresponded to the quality level P^* – the best quality level, as shown in Figure 1.

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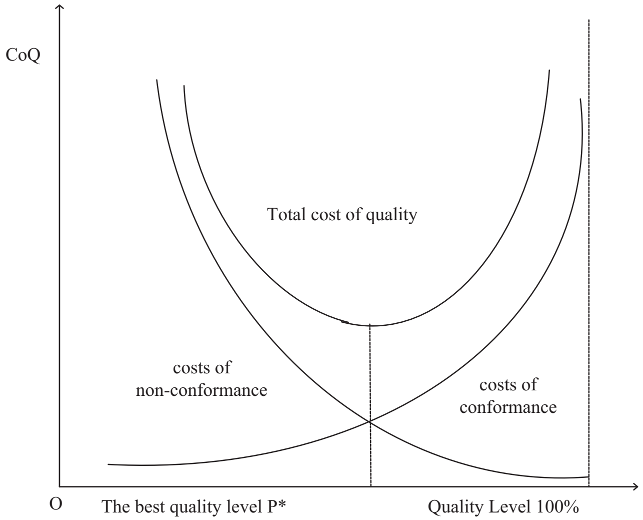

Figure 1.
Juran's quality cost characteristic curve

On the basis of the PAF model and Juran's quality characteristic curve, other scholars have put forward the optimal exponential function model for researching cost optimisation, the best quality cost model based on Taguchi's loss function, the cost optimisation model based on K.K. Govil's function, the best quality cost model based on the Cobb-Douglas production function, etc. These have played a huge role in promoting the relationship between quality cost and quality management levels.

All of these models have a large number of unknown parameters and the parameters in these models should be estimated according to regression methodology. When there is no more information and there are no more data about quality cost and quality level in practical application, all of these models cease to work. In this paper, we first discuss the modelling mechanism of several original cost control models, and then propose a new method for forecasting quality costs based on a discrete grey model (DGM). In order to eliminate the perturbation of a system's behavioural data and improve the prediction accuracy of the model, we have introduced the weakening buffer operator to weaken the time series of quality cost and reduce its randomness. The DGM model and the exponential function model have also been compared in the context of a real case to verify their feasibility and rationality.

2. Quality cost measurement models

2.1 The optimal exponential function model

The exponential function model simulates the trend of costs of conformance and costs of non-conformance by using a growth curve (also called an S-curve, including elements of the PEAR model, the logistic model and the Gong Paz model). Prevention cost and appraisal cost are known as costs of conformance and can be expressed as a positive exponential function y_1, while internal failure costs and external failure costs are also referred to as costs of non-conformance and can be represented by a negative exponential function y_2. Therefore, the exponential function model of total mass cost is:

$$y = y_1 + y_2 = a_1 e^{b_1 P} + a_2 e^{-b_2 P} \tag{1}$$

where a_1 is the maximum cost incurred when the yield of products tends to 100 per cent; a_2 the minimum cost of waste loss incurred when the qualification rate is 100 per cent;

b_1 and b_2 the slope of the curve and parameter, respectively; and P the product quality level, generally expressed in terms of the production pass rate of products.

From the shape of the total mass characteristic cost curve, we can see that when the slope of the curve is 0, the corresponding quality level reaches its most suitable position. We can calculate:

$$P^* = \frac{\ln a_2 b_2 - \ln a_1 b_1}{b_1 + b_2} \tag{2}$$

and further we can obtain the best quality cost:

$$y^* = a_1 e^{b_1 P^*} + a_2 e^{-b_2 P^*} \tag{3}$$

2.2 The best quality cost model based on Taguchi's loss function

According to the quality management expert Genichi, the concept of quality cost is the total effect of quality input and quality loss. It needs to consider not only recent investments, but also long-term economic benefits, with P expressing product qualification rate, $C(P)$ representing total CoQ, and $C_1(P)$ and $C_2(P)$ representing costs of non-conformance and costs of conformance, respectively. From the mass cost characteristic curve, when the product qualification rate $P = 100$ per cent, the quality of costs of failure reaches its smallest, recorded as a constant a_1. The costs of non-conformance at $P = 1$, according to Taylor's formula, can be expanded to:

$$C_1(P) = C_1(1) + C_1'(1)(P-1) + \frac{C_1''(1)}{2!}(P-1)^2 + o\left[(P-1)^2\right].$$

Since the costs of non-conformance are a minimum a_1 when $P = 100$ per cent, so $C_1'(1) = 0$. Let $b_1 = C_1''(1)/2!$, as high-order infinitesimal items can be ignored, and the costs of non-conformance are approximately:

$$C_1(P) = a_1 + b_1(P-1)^2.$$

Costs of conformance include prevention cost A and appraisal cost B, that is $C_2(P) = A + B$. The relationship between product qualification rate P and A, B can be expressed by the Cobb-Douglas production function, namely $P = KA^\alpha B^\beta$. Taking the Lagrange function $L(A, B, \mu)$, we can get:

$$L(A, B, \mu) = KA^\alpha B^\beta + \mu(C_2 - A - B)$$

Let $(\partial L)/(\partial A) = 0$, $(\partial L)/(\partial B) = 0$, $(\partial L)/(\partial \mu) = 0$, and we can calculate $C_2(P) = a_2 P^{b_2}$, where:

$$a_2 = \left[A\left(\frac{\alpha}{\alpha+\beta}\right)^\alpha \left(\frac{\alpha}{\alpha+\beta}\right)^\beta\right]^{-\frac{1}{\alpha+\beta}}, \quad b_2 = \frac{1}{\alpha+\beta}$$

The best quality cost model based on Taguchi's loss function is:

$$C(P) = C_1(P) + C_2(P) = a_1 + b_1(P-1)^2 + a_2 P^{b_2}$$

Among them, the parameters a_1, a_2, b_1 and b_2 have the same meaning as above. In this situation, the most appropriate quality level is not necessarily at the intersection point of costs of conformance and costs of non-conformance, and it can be regarded as an improvement of Juran's quality characteristics curve, which is shown in Figure 2.

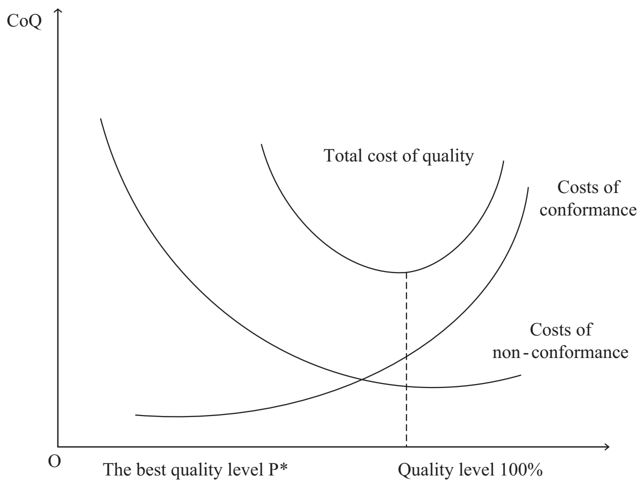

Figure 2.
The best quality cost model based on Taguchi's loss function

From Figure 2, we can see that the lowest quality level is no longer at the intersection of costs of conformance and costs of non-conformance. As a matter of fact, the best quality cost model based on Taguchi's loss function is more authentic.

2.3 The cost optimisation model based on K.K. Govil's function

Using K.K. Govil's (1984) function to establish the mathematical model of mass cost, we mainly considered the influence of pass rate on quality cost and used K.K. Govil's function to simulate the costs of conformance and the costs of non-conformance. As the costs of conformance can be expressed by $C_1 = K((P)/(1-P))^r$ and the costs of non-conformance can be denoted by $C_2 = F((1-P)/(P))^g$, then the total mass cost model is:

$$C = C_1 + C_2 = K\left(\frac{P}{1-P}\right)^r + F\left(\frac{1-P}{P}\right)^g$$

In which P is the product qualification rate, K is the loss caused by each scrap, and F is the coefficient of failure cost change with the ratio of product failure rate to product qualification rate. From the shape of the total mass characteristic cost curve, we can see that when $(dC)/(dP) = 0$, the corresponding quality level reaches its most suitable quality level P^*. Then we can obtain:

$$P^* = \frac{1}{1 + \left(\frac{Kr}{Fg}\right)^{\frac{1}{g+r}}} \tag{4}$$

and further we can obtain the best quality cost:

$$C^* = K\left(\frac{P^*}{1-P^*}\right)^r + F\left(\frac{1-P^*}{P^*}\right)^g \tag{5}$$

2.4 The best quality cost model based on the Cobb-Douglas production function

The optimal cost model based on the Cobb-Douglas production function avoids the contingent and stochastic factors in the statistical process. The costs of non-conformance are expressed by $C_1 = a_1 P^{-\beta_1} e^{\varepsilon}$, and the costs of conformance are expressed by $C_2 = a_2 P^{\beta_2} e^{\varepsilon}$. Then the total cost model is established as follows:

$$C = C_1 + C_2 = a_1 P^{-\beta_1} e^{\varepsilon} + a_2 P^{\beta_2} e^{\varepsilon}$$

where P is the product qualification rate, a_1, a_2, β_1, and β_2 the parameters to be estimated and $\beta_1 > 0$, $\beta_2 > 0$. ε the error term, reflecting the effects of contingencies and random factors.

From the shape of the total mass characteristic cost curve, we can see that when $(dC)/(dP) = 0$, the corresponding quality level reaches its most suitable quality level P^*. Then we can obtain:

$$P^* = \left(\frac{a_1 \beta_1}{a_2 \beta_2}\right)^{\frac{1}{\beta_1 + \beta_2}} \tag{6}$$

and further we can obtain the best quality cost:

$$C^* = a_1 P^* - \beta_1 e^{\varepsilon} + a_2 P^* \beta_2 e^{\varepsilon} \tag{7}$$

3. The discrete grey forecasting model

Grey system theory has been used widely in industry, agriculture, energy, transportation and other fields after 30 years of development, and has successfully solved a lot of practical problems in production, life and scientific research (Liu *et al.*, 2015). The grey prediction model is an important part of grey system theory. Based on a small amount of system behaviour data, the grey prediction model can be formed, and based on the excavation of some known information, the grey prediction model can be used to predict the system.

Assume that the time series of quality cost is $X = (x(1), x(2), \ldots, x(n))$. As historical quality cost data often suffers from a certain amount of interference, if we build the grey system model for cost analysis and prediction directly, the actual application process will inevitably appear to be inaccurate and less representative of the true situation. In order to eliminate any perturbation of a system's behavioural data effectively, we can introduce the weakening buffer operator to weaken the time series of quality cost and reduce its randomness, so as to improve the prediction accuracy of the model:

Theorem 3.1. Assume $X = (x(1), x(2), \ldots, x(n))$ is a data sequence of a system's behavioural characteristic, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the corresponding weight vector, where $\omega_i > 0$, $i = 1, 2, \ldots, n$, let $XD = (x(1)d, x(2)d, \ldots, x(n)d)$, where D is defined as follows (Liu *et al.*, 2017a):

$$\begin{aligned} x(k)d &= x(k)\left[x(k)/\frac{\omega_k x(k) + \omega_{k+1} x(k+1) + \cdots + \omega_n x(n)}{\omega_k + \omega_{k+1} + \cdots + \omega_n}\right]^{\alpha} \\ &= x(k)\left[x(k)/\frac{\sum_{i=k}^{n} \omega_i x(i)}{\sum_{i=k}^{n} \omega_i}\right]^{\alpha} \end{aligned} \tag{8}$$

When $\alpha < 0$, D is always a weakening buffer operator, whether X is a monotonic increasing a sequence, decreasing a sequence or not.

The weakening buffer operators include an average weakening buffer operator (AWBO), a weighted AWBO and a weighted geometric AWBO and so on. Here, it is assumed that there is no difference in the weight of quality cost at each time point, and that the randomness of the original quality cost data sequence can be weakened by an AWBO. These weakening buffer operators can be selected according to actual situations.

The time series, after being weakened by the first-order operator, is:

$$XD = (x(1)d, x(2)d, \cdots, x(n)d)$$

$$x(k)d = \frac{1}{n-k+1}[x(1)+x(2)+\cdots+x(n)]$$

The time series, after being weakening by the second-order operator, is:

$$XD^2 = \left(x(1)d^2, x(2)d^2, \cdots, x(n)d^2\right)$$

$$x(k)d^2 = \frac{1}{n-k+1}[x(1)d+x(2)d+\cdots+x(n)d]$$

The cost sequence after the weakening of the practical buffer operator can effectively eliminate the stochastic influence factors, and the problem can be analysed by the grey system series forecasting model. By using $XD^2 = (x(1)d^2, x(2)d^2, \ldots, x(n)d^2)$ as an initial sequence, the first-order accumulating generation operator, $X_i^{(1)} = (x_i^{(1)}(1), x_i^{(1)}(2), \cdots, x_i^{(1)}(n))$ can be obtained thereafter, in which$x_i^{(1)}(k) = \sum_{k=1}^{n} x(k)d^2$.

Construct differential equation $x_i^{(1)}(k+1) = \beta_1 x_i^{(1)}(k) + \beta_2$, and its general solution is:

$$x_i^{(1)}(k) = C\beta_1^{k-1} + \frac{\beta_2}{1-\beta_1}, k = 1, 2, \cdots, n$$

where C is any constant, which can be determined according to initial conditions.

When $k=1$, let $x_i^{(1)}(1) = x(1)d^2$, then $C = x(1)d^2 - (\beta_2)/(1-\beta_1)$. Let the solution of the differential equation be the time response, and thus the DGM prediction function of quality cost is:

$$\hat{x}_i^{(1)}(k) = \left(x(1)d^2 - \frac{\beta_2}{1-\beta_1}\right)\beta_1^{k-1} + \frac{\beta_2}{1-\beta_1} \tag{9}$$

The parameter sequence is $\hat{\beta} = [\beta_1, \beta_2]^T$, and it satisfies $\hat{\beta} = (B^T B)^{-1} B^T Y$, in which:

$$B = \begin{bmatrix} x_i^{(1)}(1) & 1 \\ x_i^{(1)}(2) & 1 \\ \cdots & \cdots \\ x_i^{(1)}(n-1) & 1 \end{bmatrix}, Y = \begin{bmatrix} x_i^{(1)}(2) \\ x_i^{(1)}(3) \\ \cdots \\ x_i^{(1)}(n) \end{bmatrix}$$

4. Case study

In order to verify the validity of the discrete grey prediction model, this paper uses a real example from the extant literature (Zhi, 2007) to compare the DGM model with the traditional exponential function model. Through the calculation of cost data and a summary of a company's products over a period of time (equal time points), the relevant information about the CoQ and quality level is shown in Table I.

First, we used the discrete grey prediction model in part 3 to forecast the fluctuation trend of quality cost. To start with, we can establish a quality cost time series.

$X = (13.45, 12.9, 12.37, 12.41, 13.18, 13.73)$, then, operating a weakening buffer operator to X, the time series after the weakening buffer operator is:

$$X_i^{(0)} = (13.19, 13.23, 13.3, 13.43, 13.59, 13.73)$$

Building the discrete grey forecasting model, according to sequence $X_i^{(0)}$, we can get the time response function:

$$\hat{x}_i^{(1)}(k) = 1362.26 \cdot 1.009689^{k-1} - 1349.07$$

Then we can calculate the quality cost at different times, and thus the simulation error of the discrete grey forecasting model is presented in Table II.

The sum of squares of residuals is $s = \varepsilon\varepsilon^T = 0.002$, and the mean of the relative error is $\overline{\Delta}_1 = \frac{1}{6}\sum_{k=1}^{6} \Delta_k = 0.118\%$.

The exponential function model established in the literature (Zhi, 2007) is $C(P) = 1.83 \cdot 6.58^P + 30.16 \cdot 0.09^P$. The simulation error of the exponential function model is presented in Table III.

The sum of squares of residuals is $s = \varepsilon\varepsilon^T = 0.416$, and the mean of the relative error is $\overline{\Delta}_2 = \frac{1}{6}\sum_{k=1}^{6} \Delta_k = 1.689\%$.

Based on the comparison of the squares of residuals and the average relative error of these two models, the prediction accuracy of the DGM after being weakened by the buffer operator has been improved obviously compared with the exponential function model, which can be used to optimise the quality cost, so there is a certain degree of feasibility and rationality.

5. Conclusion

More and more enterprises are beginning to realise the importance of quality cost in quality control. Different quality cost prediction optimisation models can quantitatively forecast and simulate quality costs at different quality management levels. When there is a lack of information, a DGM can be used to predict the fluctuation trend of quality cost based on

Table I. Quality cost data over a period of time

Quality level	0.61	0.69	0.74	0.84	0.89
Costs of conformance	5.83	6.29	7.21	8.86	10.17
Costs of non-conformance	7.07	6.08	5.2	4.32	3.56
Total cost of quality	12.9	12.37	12.41	13.18	13.73

Table II. The simulation error of DGM

Serial number	1	2	3	4	5	6
Raw data x_i	13.19	13.23	13.3	13.43	13.59	13.73
Simulation data $\hat{x}_i$	13.19	13.199	13.327	13.456	13.586	13.718
Residual error ε	0	0.027	0.023	0.025	0.006	0.012
Relative error Δ	0	0.208%	0.175%	0.189%	0.045%	0.088%

Table III. The simulation error of the exponential function model

Quality level P	0.56	0.61	0.69	0.74	0.84	0.89
Raw data x_i	13.45	12.9	12.37	12.41	13.18	13.73
Simulation data $\hat{x}_i$	13.087	12.718	12.441	12.455	12.898	13.325
Residual error ε	−0.363	−0.182	0.071	0.045	−0.282	−0.405
Relative error Δ	2.700%	1.413%	0.572%	0.359%	2.141%	2.949%

Juran's quality cost characteristic curve. What's more, the randomness of time series can be weakened by a weakening buffer operator before modelling, which can effectively reduce the factor perturbation and reduce the randomness of the data.

The purpose of the forecast is to control the quality of the enterprise better, and the effective control and management of the cost is an important guarantee for normal capital flows. Cost controls throughout the operation of the enterprise have always been implemented to establish a long and effective accounting system to strengthen scientific and rational cost accounting. At the same time, in the process of cost forecasting and control, we must consider all aspects of uncertainty derived from unknown factors, which is the only way to make the forecast more feasible for the quality of enterprise management to provide more practical guidance.

With the rise of "zero defect" management and total quality management (Dong *et al.*, 2017), most companies are no longer using a single quality cost minimum target for quality control and management, but a large number of external factors should be taken into account by quality of cost models when establishing the actual dynamic cost forecasting model of an enterprise. The optimal quality level obtained by Juran's quality cost curve is required to accept an unqualified product rate for the firm. With the passage of time and the improvement of management methods, the limitations of this quality cost model have become more and more obvious. In these circumstances, we should use new theories to guide quality cost management in enterprises.

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